Study Guide and Review

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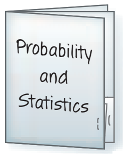
Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

The Counting Principle, Permutations, and Combinations (Lessons 12-1 and 12-2)

- Fundamental Counting Principle: If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then event *M* followed by event *N* can occur in $m \cdot n$ ways.
- Permutation: order of objects is important.
- Combination: order of objects is not important.

Probability (Lessons 12-3 and 12-4)

- Two independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Two dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$
- Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
- Inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Statistical Measures (Lesson 12-5)

- To represent a set of data, use the mean if the data are spread out, the median when the data has outliers, or the mode when the data are tightly clustered around one or two values.
- Standard deviation for *n* values: $\bar{x}$ is the mean,

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

The Normal Distribution (Lesson 12-6)

- The graph is maximized at the mean and the data are symmetric about the mean.

Binomial Experiments, Sampling, and Error (Lessons 12-7 and 12-8)

- A binomial experiment exists if and only if there are exactly two possible outcomes, a fixed number of independent trials, and the possibilities for each trial are the same.

Key Vocabulary

binomial distribution (p. 730)
binomial experiment (p. 730)
binomial probability (p 731)
combination (p. 692)
compound event (p. 710)
dependent events (p. 686)
event (p. 684)
exponential distribution (p 729)
exponential probability (p. 729)
inclusive events (p. 712)
independent events (p. 684)
measure of variation (p. 718)
mutually exclusive events (p. 710)
normal distribution (p. 724)
outcome (p. 684)
permutation (p. 690)
probability (p. 697)
probability distribution (p. 699)
random (p. 697)
random variable (p. 699)
relative-frequency histogram (p. 699)
sample space (p. 684)
simple event (p. 710)
standard deviation (p. 718)
unbiased sample (p. 741)
uniform distribution (p. 699)
univariate data (p. 717)
variance (p. 718)

Vocabulary Check

Choose the term that best matches each statement or phrase. Choose from the list above.

1. the ratio of the number of ways an event can succeed to the number of possible outcomes
2. an arrangement of objects in which order does not matter
3. two or more events in which the outcome of one event affects the outcome of another event
4. a function that is used to predict the probabilities of an event based on time
5. two events in which the outcome can never be the same
6. an arrangement of objects in which order matters
7. the set of all possible outcomes
8. an event that consists of two or more simple events

Lesson-by-Lesson Review

12-1 The Counting Principle (pp. 684–689)

9. PASSWORDS The letters a, c, e, g, i, and k are used to form 6-letter passwords. How many passwords can be formed if the letters can be used more than once in any given password?

Example 1 How many different license plates are possible with two letters followed by three digits?

There are 26 possibilities for each letter. There are 10 possibilities, the digits 0–9, for each number. Thus, the number of possible license plates is as follows.

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$ or 676,000

12-2 Permutations and Combinations (pp. 690–695)

10. A committee of 3 is selected from Jillian, Miles, Mark, and Nikia. How many committees contain 2 boys and 1 girl?

11. Five cards are drawn from a standard deck of cards. How many different hands consist of four queens and one king?

12. A box of pencils contains 4 red, 2 white, and 3 blue pencils. How many different ways can 2 red, 1 white, and 1 blue pencil be selected?

Example 2 A basket contains 3 apples, 6 oranges, 7 pears, and 9 peaches. How many ways can 1 apple, 2 oranges, 6 pears, and 2 peaches be selected?

This involves the product of four combinations, one for each type of fruit.

$C(3, 1) \cdot C(6, 2) \cdot C(7, 6) \cdot C(9, 2)$

$= \frac{3!}{(3-1)!1!} \cdot \frac{6!}{(6-2)!2!} \cdot \frac{7!}{(7-6)!6!} \cdot \frac{9!}{(9-2)!2!}$

$= 3 \cdot 15 \cdot 7 \cdot 36$ or 11,340 ways

12-3 Probability (pp. 697–702)

13. A bag contains 4 blue marbles and 3 green marbles. One marble is drawn from the bag at random. What is the probability that the marble drawn is blue?

14. COINS The table shows the distribution of the number of heads occurring when four coins are tossed. Find $P(H = 3)$.

H = Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Example 3 A bag of golf tees contains 23 red, 19 blue, 16 yellow, 21 green, 11 orange, 19 white, and 17 black tees. What is the probability that if you choose a tee from the bag at random, you will choose a green tee?

There are 21 ways to choose a green tee and 23 + 19 + 16 + 11 + 19 + 17 or 105 ways not to choose a green tee. So, s is 21 and f is 105.

$P(\text{green tee}) = \frac{s}{s + f}$

$= \frac{21}{21 + 105}$ or $\frac{1}{6}$

12–4 Multiplying Probabilities (pp. 703–709)

Determine whether the events are *independent* or *dependent*. Then find the probability.

15. Two dice are rolled. What is the probability that each die shows a 4?

16. Two cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart and a club in that order.

17. Luz has 2 red, 2 white, and 3 blue marbles in a cup. If she draws two marbles at random and does not replace the first one, find the probability of a white marble and then a blue marble.

Example 4 **There are 3 dimes, 2 quarters, and 5 nickels in Langston's pocket. If he reaches in and selects three coins at random without replacing any of them, what is the probability that he will choose a dime d, then a quarter q, and then a nickel n?**

Because the outcomes of the first and second choices affect the later choices, these are dependent events.

$P(d, \text{ then } q, \text{ then } n) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{5}{8}$ or $\frac{1}{24}$

The probability is $\frac{1}{24}$ or about 4.2%.

12–5 Adding Probabilities (pp. 710–715)

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

18. A die is rolled. What is the probability of rolling a 6 or a number less than 4?

19. A die is rolled. What is the probability of rolling a 6 or a number greater than 4?

20. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a red card?

21. There are 5 English, 2 math, and 3 chemistry books on a shelf. If a book is randomly selected, what is the probability of selecting a math book or a chemistry book?

Example 5 **Trish has four \$1 bills and six \$5 bills. She takes three bills from her wallet at random. What is the probability that Trish will select at least two \$1 bills?**

$P(\text{at least two } \$1)$

$= P(\text{two } \$1, \$5) + P(\text{three } \$1, \text{no } \$5)$

$= \frac{C(4, 2) \cdot C(6, 1)}{C(10, 3)} + \frac{C(4, 3) \cdot C(6, 0)}{C(10, 3)}$

$= \frac{\frac{4! \cdot 6!}{(4-2)!2!(6-1)!1!}}{C(10, 3)} + \frac{\frac{4! \cdot 6!}{(4-3)!3!(6-0)!0!}}{C(10, 3)}$

$= \frac{36}{120} + \frac{4}{120}$ or $\frac{1}{3}$

The probability is $\frac{1}{3}$ or about 33%.

CHAPTER 12

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12-6 Statistical Measures (pp. 717–723)

FOOD **For Exercises 22 and 23, use the frequency table that shows the number of dried apricots per box.**

Apricot Count	Frequency
19	1
20	3
21	5
22	4

22. Find the mean, median, mode, and standard deviation of the apricots to the nearest tenth.

23. For how many boxes is the number of apricots within one standard deviation of the mean?

Example 6 **Find the variance and standard deviation for {100, 156, 158, 159, 162, 165, 170, 190}.**

Step 1 Find the mean.

$$\frac{100 + 156 + 158 + 159 + 162 + 165 + 170 + 190}{8}$$

$$= \frac{1260}{8} \text{ or } 157.5$$

Step 2 Find the standard deviation.

$$\sigma^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}$$

$$\sigma^2 = \frac{(100 - 157.5)^2 + \ldots + (190 - 157.5)^2}{8}$$

$$\sigma^2 = \frac{4600}{8}$$ Simplify.

$$\sigma^2 = 575$$ Divide.

$$\sigma \approx 23.98$$ Take the square root of each side.

12-7 The Normal Distribution (pp. 724–728)

UTILITIES **For Exercises 24–27, use the following information.**

The utility bills in a city of 5000 households are normally distributed with a mean of \$180 and a standard deviation of \$16.

24. About how many utility bills were between \$164 and \$196?

25. About how many bills exceeded \$212?

26. About how many bills were under \$164?

27. What is the probability that a random bill is between \$164 and \$180?

BASEBALL **For Exercises 28 and 29, use the following information.**

The average age of a major league baseball player is normally distributed with a mean of 28 and a standard deviation of 4 years.

28. About what percent of major league baseball players are younger than 24?

29. If a team has 35 players, about how many are between the ages of 24 and 32?

Example 7 **Mr. Byrum gave an exam to his 30 Algebra 2 students at the end of the first semester. The scores were normally distributed with a mean score of 78 and a standard deviation of 6.**

a. What percent of the class would you expect to have scored between 72 and 84?
Since 72 and 84 are 1 standard deviation to the left and right of the mean, respectively, 34% + 34% or 68% of the students scored within this range.

b. What percent of the class would you expect to have scored between 90 and 96?
90 to 96 on the test includes 2% of the students.

c. Approximately how many students scored between 84 and 90?
84 to 90 on the test includes 13.5% of the students; $0.135 \times 30 = 4$ students.

Mixed Problem Solving
For mixed problem-solving practice, see page 937.

12-8 Exponential and Binomial Distribution (pp. 729-733)

30. The average person has a pair of automobile windshield wiper blades for 6 months. What is the probability that a randomly selected automobile has a pair of windshield wiper blades older than one year?

LAWS For Exercises 31 and 32, use the following information.

A polling company wants to estimate how many people are in favor of a new environmental law. The polling company polls 20 people. The probability that a person is in favor of the law is 0.5.

31. What is the probability that exactly 12 people are in favor of the new law?

32. What is the expected number of people in favor of the law?

Example 8 According to a recent survey, the average teenager spends one hour a day on an outdoor activity. What is the probability that a randomly selected teenager spends more than 1.5 hours per day outside?

Use the first exponential distribution function. The mean is 1, and the inverse of the mean is 1.

$f(x) = e^{-mx}$ Exponential Distribution Function

$= e^{-1(1.5)}$ Replace m with 1 and x with 1.5.

$= e^{-1.5}$ Simplify.

≈ 0.2231 or 22.31% Use a calculator.

There is a 22.31% chance that a randomly selected teenager spends more than 1.5 hours a day outside.

12-9 Binomial Experiments (pp. 735–739)

Find each probability if a number cube is rolled twelve times.

33. P(twelve 3s) **34.** P(exactly one 3)

35. WORLD CULTURES The Cayuga Indians played a game of chance called *Dish*, in which they used 6 flattened peach stones blackened on one side. They placed the peach stones in a wooden bowl and tossed them. The winner was the first person to get a prearranged number of points. Assume that each face (black or neutral) of each stone has an equal chance of showing up. Find the probability of each possible outcome.

Example 9 To practice for a jigsaw puzzle competition, Laura and Julian completed four jigsaw puzzles. The probability that Laura places the last piece is $\frac{3}{5}$, and the probability that Julian places the last piece is $\frac{2}{5}$. What is the probability that Laura will place the last piece of at least two puzzles?

$P = L^4 + 4L^3J + 6L^2J^2$

$= \left(\frac{3}{5}\right)^4 + 4\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right) + 6\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$

$= \frac{81}{625} + \frac{216}{625} + \frac{216}{625}$ or 0.8208

The probability is about 82%.

Study Guide and Review

12–10 Sampling and Error (pp. 741–744)

36. **ELECTION** According to a poll of 300 people, 39% said that they favor Mrs. Smith in an upcoming election. What is the margin of sampling error?

37. **FREEDOMS** In a poll asking people to name their most valued freedom, 51% of the randomly selected people said it was the freedom of speech. Find the margin of sampling error if 625 people were randomly selected.

38. **SPORTS** According to a recent survey of mothers with children who play sports, 63% of them would prefer that their children not play football. Suppose the margin of error is 4.5%. How many mothers were surveyed?

Example 10 **In a survey taken at a local high school, 75% of the student body stated that they thought school lunches should be free. This survey had a margin of error of 2%. How many people were surveyed?**

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$ Margin of sampling error

$$0.02 = 2\sqrt{\frac{0.75(1-0.75)}{n}}$$ $ME = 0.02$, $p = 0.75$

$$0.01 = \sqrt{\frac{0.75(1-0.75)}{n}}$$ Divide each side by 2.

$$0.0001 = \frac{0.75(0.25)}{n}$$ Square each side.

$$n = \frac{0.75(0.25)}{0.0001}$$ Multiply and divide.

$$= 1875$$ Simplify.

There were about 1875 people in the survey.